10 Mathematics

Theory and Exercises on Linear Inequalities

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1 - Meaning of the term ‘linear Inequality’

What do we mean by

Linear[1] Inequality[2] ?

1 – Linear = Referring to a linear function, meaning a function that when graphed, results in a straight line.

> A linear function is any function that could take the form $y = mx + c$.

2 – Inequality = A mathematical statement that compares two numbers which *are not equal.*

The opposite of an inequality is an equation, which you are very familiar with:

> For the two terms *a* and *b,* an *equation* would be $a = b$, whereas an *inequality* would be $a > b$ or $a \leq b$.

$a = b$ is an *equation* because it is a statement that compares two terms which are equal, whereas $a > b$ is an inequality because it compares a larger term with a smaller term (unequal terms).

2 – Three forms for representing linear inequalities

We will examine three forms for representing statements that compare unequal numbers (linear inequalities).

Let us express the following linear inequality in three different forms.

$$\textit{The sum of two lots of } x \textit{ and three is greater than seven.}$$

1. Inequality notation

   This is the most convenient form to start with, and it uses the inequality symbols $<, >, \leq$ and $\geq$ to express the unequal comparison:

$$2x + 3 > 7$$

   i. $'two\ lots\ of\ x'$ means $two\ times\ x$ or $2x$

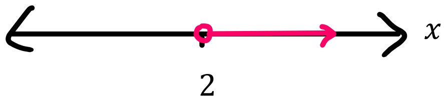
   ii. $'The\ sum\ of\ (2x)\ and\ three'$ means $2x + 3$

   Question for you, how could I express the statement

   $'the\ sum\ of\ five\ and\ three\ lots\ of\ x\ is\ less\ than\ forty'$ in *inequality notation*?

2. Number line

This form uses a one dimensional line to express the unequal comparison.



However, to use this form, we must first make our variable (e.g. x) the *subject* of our inequality by *solving the linear inequality.* We will discuss the *solving of inequalities* in more depth later.

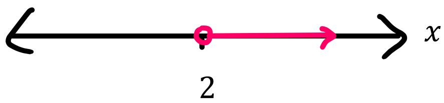
$$2x + 3 > 7$$

$$2x + 3 - 3 > 7 - 3$$

$$2x \div 2 > 4 \div 2$$

$$\therefore x > 2$$

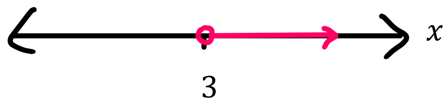
Our solved linear inequality is $x\ is\ greater\ than\ two$, which we can now simply call an inequality, and is represented on the number line as



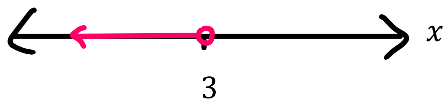
There are various rules for representing inequalities on number lines. For the variable x,

a. If we want to say that x is *greater than* or *less than* a value, we draw an open circle (indicating that we are *excluding* the value).

$$x > 3$$

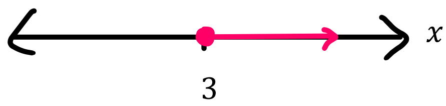


$$x < 3$$

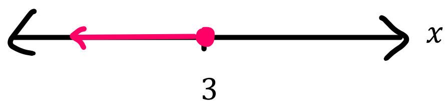


b. If we want to say that x is *greater than or equal to* or *less than or equal to* a value, we draw a closed circle (meaning that we are *including* the value).

$$x \geq 3$$

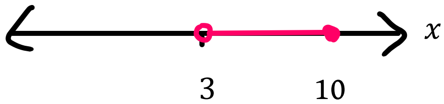


$$x \leq 3$$



c. *Bounds* are what we can call the values which we are saying that x is *greater than/less than (or equal to)*. For instance, if

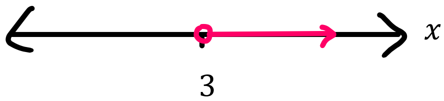
$$3 < x \leq 10$$



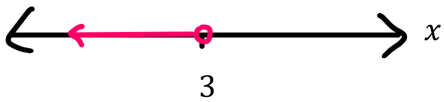
then we can call 3 our *lower bound,* and 10 our *upper bound.* We call 10 the 'upper' bound because it is the highest value that x can be, and we call 3 our lower bound because x, whilst not being equal to, cannot be less than 3.

d. You may have noticed in (a) and (b) that our inequalities only had either upper or lower bounds, not both.

$$x > 3$$



$$x < 3$$



When this is the case, we can represent the missing bound as an arrow. This arrow indicates that x could be any value extending towards *positive or negative infinity.*

Question for you, how could I express the statement *'the sum of five and three lots of x is less than forty'* using a number line?

3. Interval notation

This third form is the most concise of the three, and

$$it\ expresses\ the\ inequality\ as\ an\ interval^3.$$

3 – Interval = A range of real numbers (meaning numbers that could fill all the points on an infinite, continuous number line) between two *bounds.*

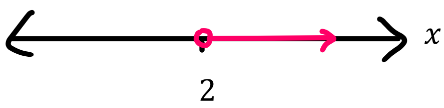
If we recall that the example linear inequality that we are working with is

$$the\ sum\ of\ two\ lots\ of\ x\ and\ three\ is\ greater\ than\ seven.$$

Which we can express in (1) *inequality notation* and using (2) a *number line* as

(1) $2x + 3 > 7$

(2) $x > 2$



Also recall (page 5) that we had to *solve* the linear inequality for x in order to express it using (2) a number line. This $solving\ for\ x$ gave us $x > 2$.

In order to express our example linear equality using (3) *interval notation*, we also need to have $solved$ it for x, which we have conveniently already done.

We can therefore express our example linear inequality using *interval notation* as

$$[2, \infty)$$

Very concise indeed!

As we did for the *number line* form, we will examine the rules for expressing inequalities in interval notation. Remember, our linear inequality needs to first be *solved for x* before we can express it in *interval notation.*

1. If we only have either an *upper bound* or a *lower bound*, but not both, we express the missing bound as either *positive or negative infinity*.

   If we only have a *lower bound*:

   $$x > 2$$

   $$\therefore (2, \infty)$$

   If we only have an *upper bound*:

   $$x < 2$$

   $$\therefore (-\infty, 2)$$

2. If we want to say that x is *greater than* or *less than* a value, we write a *curvy bracket* adjacent to that value (notice that we don't need to write the x when expressing the inequality in *interval notation*).

$$-1 < x \leq 2$$

$$\therefore (-1, 2]$$

3. If we want to say that x is *greater than or equal to* or *less than or equal to* a value, we write a *square bracket* adjacent to that value.

$$-1 < x \leq 2$$

$$\therefore (-1, 2]$$

Question for you, how could I express the statement

$'the\ sum\ of\ five\ and\ three\ lots\ of\ x\ is\ less\ than\ forty'$ using a interval notation?

3 – Solving linear inequalities

We can *solve linear inequalities* in (1) *inequality notation* using the *inverse operation* conventions that we use to solve *linear equations* – which I presume that you are familiar with.

Using *inverse operations* to solve

a linear equation	a linear inequality
$2x + 3 = 7$	$2x + 3 > 7$
$2x = 4$ $\{subtract\ 3\ from\ both\ sides\}$	$2x > 4$ $\{subtract\ 3\ from\ both\ sides\}$
$x = 2$ $\{divide\ both\ sides\ by\ 2\}$	$x > 2$ $\{divide\ both\ sides\ by\ 2\}$

As you can see, in the above case the conventions are identical.

But the only nuance occurs when we are required to

$$multiply\ or\ divide\ both\ sides\ of\ an\ inequality\ by\ a\ negative\ number:$$

Linear equation	Linear inequality
$-2x + 3 = 7$	$-2x + 3 > 7$
$-2x = 4 \quad \{subtract\ 3\ from\ both\ sides\}$	$-2x > 4 \quad \{subtract\ 3\ from\ both\ sides\}$
$\therefore x = -2 \quad \{divide\ both\ sides\ by - 2\}$	$\therefore x < -2 \quad \{divide\ both\ sides\ by - 2\}$

If we $multiply\ or\ divide$ both sides of an inequality by a $negative\ number$, we must $flip\ the\ inequality\ symbol$, as in the below examples

1	2
$12 - 4x < 52$	$-\frac{3x}{2} - 4 < 4$
$-4x < 40$	$-\frac{3x}{2} < 8$
$\therefore x > -10$	$\therefore x > -\frac{3}{16}$

We $flip\ the\ inequality\ symbol$ when multiplying or dividing both sides by a negative number in order to prevent our statement from becoming $false$, which occurs when we multiply or divide both sides by a negative number $without\ the\ inequality\ flip$:

Without inequality flip	$With$ inequality flip
$4 < 5$	$4 < 5$
$-4 < -5 \quad \{multiply\ both\ sides\ by - 1\}$	$-4 > -5 \quad \{multiply\ both\ sides\ by - 1\}$
$False! \quad -4$ is larger.	

With the exception of the $inequality\ flip$, we can solve linear inequalities as if they were $linear\ equations$.

4 – Exercises

Solve the following linear inequalities for x and express your result in both

1. Number Line form and
2. Interval Notation form

a. $\frac{2}{3}x - \frac{1}{4} > \frac{5}{6}$

b. $-\frac{4}{3}x - \frac{3}{5} \leq \frac{2}{9}(x + 2)$

c. $\frac{8}{6}(x - 4) - \frac{2}{10} > \frac{7}{2}(x + 5)$

d. $\frac{3}{11}(x - 2) - \frac{2}{9} \geq \frac{8}{3}(x + \frac{4}{21})$

e. $\frac{2}{5}\left(\frac{x}{\pi} - \frac{3}{8}\right) - \frac{6}{7} < \frac{3}{10}(x + \frac{11}{19})$

f. $\frac{7}{8}\left(\frac{x}{2\pi} - \frac{4}{9}\right) - \frac{2}{5} \geq \frac{9}{4}(\frac{x}{\pi+3} + \frac{16}{23})$