7 Mathematics

Theory and exercises on improper fractions and mixed numbers

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Theory with exercises

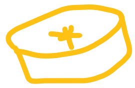
1 – Meaning of 'fraction'

When we say that a number is a *fraction*, we are saying that the number

represents a part of a quantity.

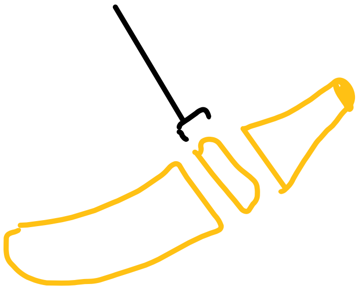
For instance, I may say that

if a banana slice



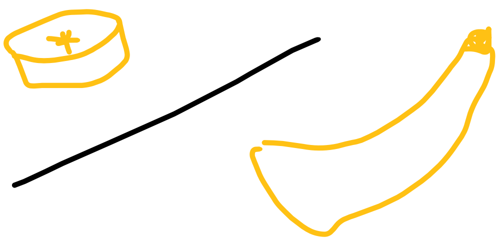
represents

a part of the quantity of a whole banana,



then

A banana slice is a fraction of a whole banana.

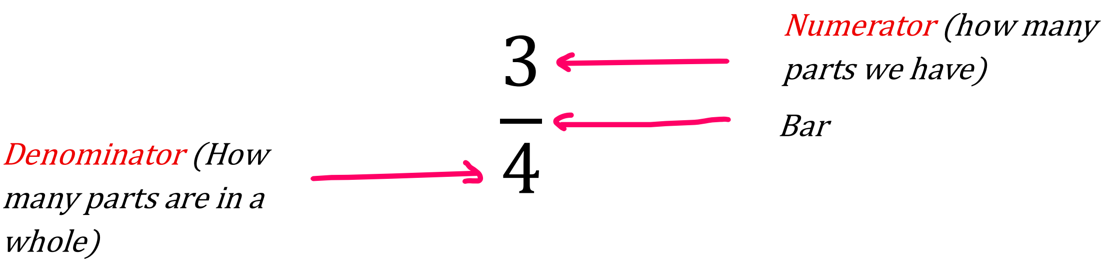


The *banana slice* is the part, and the *whole banana* is the quantity.

2 – 'Common' fractions

In our study of mathematics, we represent *fractions* as *common fractions*, which have the following components:

The common fraction



If we want to pronounce the *common fraction* $\frac{3}{4}$, we say

Three parts out of four

And, for the *common fraction* $\frac{3}{4}$

We need four parts to make a whole, but we only have three.

Question for you.

If I have sixteen marbles, and four are blue, two are yellow

three are purple, and seven are orange,

What *fraction* of my marbles are

1. Blue?
2. Yellow?
3. Purple?
4. Orange?

And can you represent these *fractions* as *common fractions*?

Now, common fractions are very useful, because we can rewrite them as the *division* of the *numerator* and *denominator* of said *common fraction*. If you remember,

$$\frac{3}{4}$$

Numerator (how many parts we have)

Denominator (How many parts are in a whole)

We can rewrite this common fraction as

$$\frac{3}{4} = 3 \div 4$$

Division symbol (replaces the bar)

Numerator (The top number of our common fraction)

Denominator (The bottom number of our common fraction)

But importantly,

$$3 \div 4 \textit{ still means three parts out of four!}$$

Questions for you. Can you write the below fractions as division expressions? And can you solve these division expressions?

a. $\frac{-50}{10}$ b. $\frac{49}{-7}$ c. $\frac{16}{4}$ d. $\frac{-121}{-11}$ e. $\frac{-144}{12}$ f. $\frac{18}{9}$ g. $\frac{24}{-6}$ h. $\frac{-63}{-9}$

3 – 'Proper' versus 'improper' fractions

So far, we have said that

$$a\ common\ fraction\ is\ expressed\ in\ the\ form\ \frac{numerator}{denominator}$$

$$for\ instance\ \frac{5}{6}\ , where\ 5\ is\ the\ numerator,$$

$$and\ 6\ is\ the\ denominator, and\ \frac{5}{6}\ means\ "five\ parts\ out\ of\ six".$$

We will now discuss *two types of common fractions*:

1 The *proper* fraction	2 The *improper* fraction
A fraction is a *proper* fraction if its *numerator* is *less than* its denominator. For instance, $\frac{5}{6}$ is a proper fraction, because its numerator (top number) is 5 and its denominator (bottom number) is 6 and 5 is less than 6. So, for $\frac{5}{6}$ the numerator is *less than* the denominator, and therefore we call it a *proper fraction.*	A fraction is an *improper* fraction if its *numerator* is *greater than* its denominator. For instance, $\frac{25}{6}$ is an improper fraction, because its numerator (top number) is 25 and its denominator (bottom number) is 6 and 25 is greater than 6. So, for $\frac{25}{6}$ the numerator is *greater than* the denominator, and therefore we call it a *improper fraction.*

Question for you. Please observe the following fractions:

a. $\frac{22}{10}$ b. $\frac{2}{-4}$ c. $\frac{1}{3}$ d. $\frac{-121}{-11}$ e. $\frac{-1}{13}$ f. $\frac{18}{8}$ g. $\frac{24}{-7}$ h. $\frac{2}{-9}$

Of all of the fractions above, what $fraction$ of them are $improper\ fractions$?

4 – Why are improper fractions called ‘improper’?

We said previously that

a fraction is improper if its numerator is greater than its denominator.

For instance, we would call $\frac{19}{9}$ an *improper fraction.*

But *why* do we call a fraction *improper* if its *numerator* is *greater than* its *denominator*? After all,

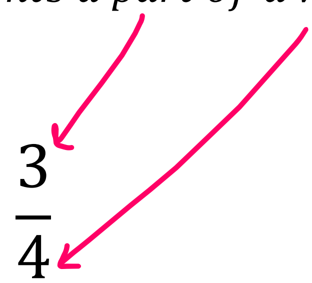
the word 'improper' means not following the rules.

So it seems, when we say that a fraction is *improper*, we are saying that it is *a fraction that is not following the rules.*

But what are the rules that these *improper fractions* are not following?

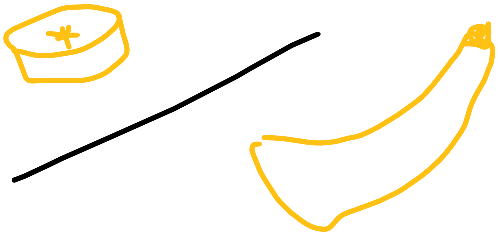
Well, if you remember our very first discussion, what a *fraction is* or *the rule of a fraction* is that:

a fraction represents a part of a whole.



For example,

A banana slice is a fraction or part of a whole banana.



But question for you,

Can 'part' of a 'whole' be bigger than the whole?

Or in other words,

Can a part of a banana be bigger than the whole banana?

It cannot! If we have a 'part' that is bigger than the whole

then it is not a part!

So, if it is true, and it is a *rule* that

a fraction is a part of a whole,

if we have a fraction like $\frac{19}{9}$, where the *part* is *bigger* than the whole,

then that fraction is breaking the rules!

And we call something that breaks the rules 'improper'

This is why we call a fraction *improper* if its *numerator* is greater than its *denominator*.

5 – Converting improper fractions to 'mixed numbers'

We said previously that $improper\ fractions$, where the $numerator$ is $greater\ than$ the denominator, are $breaking\ the\ rules$ of $fractions.$

So,

$$whenever\ we\ have\ an\ improper\ fraction,$$

$$we\ ought\ to\ get\ rid\ of\ it!$$

But how do we do so?

We do so by

$$converting\ the\ improper\ fraction\ to\ a\ 'mixed\ number'$$

But what is a $mixed\ number$?

$$A\ mixed\ number\ is\ a\ whole\ number\ written\ next\ to$$

$$a\ proper\ fraction - such\ as\ 1\frac{1}{2}$$

$$which\ we\ pronounce\ as\ "one\ and\ a\ half".$$

But how do we take an $improper\ fraction$ such as $\frac{3}{2}$, and turn it into a $mixed\ number$ such as $1\frac{1}{2}$ (one and a half)?

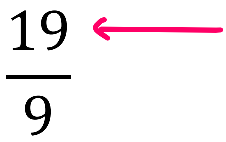
This is exactly what we will discuss next.

How to convert an improper fraction to a mixed number

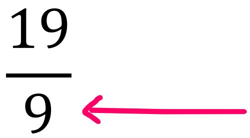
Let us use the improper fraction $\frac{19}{9}$ to demonstrate this process:

1. *Divide the numerator by the denominator, and find the remainder.*

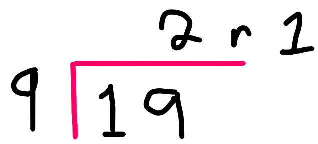
   Our *numerator* is 19, because it is on top of our fraction:



   and our *denominator* is 9, because it is on the bottom of our fraction:



   And if we divide 19 by 9 using *short division*:



   We find that $19 \div 9 = 2 \; remainder \; 1$

   (meaning that 9 can only fit completely into 19 two times, and when we do this, we are left with 1).

2. *We then use the result of our division to make the mixed fraction*:

$\frac{19}{9}$ *is our improper fraction*

19 *is our numerator*

9 *is our denominator*

1. *We divided our numerator by our denominator*

$19 \div 9 = 2 \text{ remainder } 1$

$\begin{array}{r} 2 \text{ r } 1 \\ 9 \overline{)19} \end{array}$

2 *is our whole number* (9 *fits into* 19 *two times*)

1 *is our remainder*

$So, \frac{19}{9} \text{ as a mixed number is } 2\frac{1}{9}$

i. The whole number becomes the whole number in the mixed number
ii. The remainder becomes the numerator of the proper fraction in the mixed number
iii. The denominator of the proper fraction in the mixed number stays the same as the denominator of the improper fraction that we started with.

Please attempt to convert the following *improper fractions* into *mixed fractions* using the method that we have just discussed:

a. $\frac{22}{10}$ b. $\frac{36}{7}$ c. $\frac{42}{5}$ d. $\frac{120}{7}$ e. $\frac{144}{9}$ f. $\frac{232}{3}$ g. $\frac{453}{4}$ h. $\frac{748}{5}$