Proposed solutions to quadratic identities problems

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Contents

Find r, s and t if $3(x-1)^2 + 2(x-r) + 6 = sx^2 + tx + 7$ for all x.

$$3(x-1)^2 + 2(x-r) + 6 = sx^2 + tx + 7$$

Let $x = 0$,

$3 - 2r + 6 = 7$

$-2r = -2$

$r = 1$

$$\therefore\ 3(x-1)^2 + 2(x-1) + 6 = sx^2 + tx + 7$$

$$= s[(x-1)+1]^2 + t[(x-1)+1] + 7$$

$$\{x = (x-1)+1\}$$

$$= s[(x-1)^2 + 2(x-1) + 1] + t(x-1) + t + 7$$

$$\{a^2 + b^2 = (a+b)^2 = a^2 + 2ab + b^2\}$$

$$= s(x-1)^2 + 2s(x-1) + s + t(x-1) + t + 7$$

$$3(x-1)^2 + 2(x-1) + 6 = s(x-1)^2 + (2s+t)(x-1) + (s+t+7)$$

$$= 3(x-1)^2 + (6+t)(x-1) + (10+t)$$

$$\{s = 3\}$$

$$= 3(x-1)^2 + (2)(x-1) + (6)$$

$$\left\{\begin{array}{l} 6 + t = 2 \\ \therefore t = -4 \end{array}\right\}$$

2 – Problem two

a. Find $Am^2 + B(m-1)^2 = 4m$.

b. Hence, find the sum of $1 + 3 + 5 + \cdots + 61$

$Am^2 + B(m-1)^2 = 2m - 1$

a) Find A and B

Let $m = 0$,
$B(-1)^2 = -1$
$\therefore B = -1$

Let $m = 1$
$A = 2(1) - 1$
$\therefore A = 1$

$\therefore m^2 + (m-1)^2 = 2m - 1$

b) Hence, find the sum of $1 + 3 + 5 + \ldots + 61$

{ $2m - 1$ creates the m^{th} odd number}

Let $2m - 1 = 61$
$2m = 62$
$\therefore m = 31$ {61 is the 31^{ST} odd number}

Now, $\sum_{m=1}^{31} (2m-1) = \sum_{m=1}^{31} [m^2 - (m-1)^2]$

$$= \sum_{m=1}^{31} m^2 - \sum_{m=1}^{31} (m-1)^2$$

$$= \sum_{m=1}^{31} m^2 - \sum_{m=1-1}^{31-1} [(m+1)-1]^2$$

$$= \sum_{m=1}^{31} m^2 - \sum_{m=0}^{30} m^2$$

$$= [(1)^2 + (2)^2 \ldots (31)^2] - [(0)^2 + (1)^2 \ldots (30)^2]$$

$$\therefore \sum_{m=1}^{31} (2m-1) = 31^2 = 961$$

3 – Problem three

a. Let p, q and r be distinct real numbers. Substitute three appropriate values of x in order to show that

$$\frac{(p-x)(q-x)}{(p-r)(q-r)}+\frac{(q-x)(r-x)}{(q-p)(r-p)}+\frac{(r-x)(p-x)}{(r-q)(p-q)}=1$$

for all values of x.

Let p, q, and r be 3 distinct real numbers.
Substitute 3 appropriate values of x in order to show that

$$f(x)=\underbrace{\overset{1}{\frac{(p-x)(q-x)}{(p-r)(q-r)}}}_{\text{this is a quadratic, because } (p-x)(q-x) \text{ will give you } Ax^2}+\underbrace{\overset{2}{\frac{(q-x)(r-x)}{(q-p)(r-p)}}}_{\text{so is this}}+\underbrace{\overset{3}{\frac{(r-x)(p-x)}{(r-q)(p-q)}}}_{\text{and this}}=1$$

and if you add 3 quadratics together, you still get a quadratic, so the whole left hand side could be expressed as Ax^2+Bx+C somehow.

1. Let $x=p$

$$f(p)=0+\frac{(q-p)(r-p)}{(q-p)(r-p)}+0=1$$

$\therefore f(p)=1$

2. Let $x=q$

$$f(q)=0+0+\frac{(r-q)(p-q)}{(r-q)(p-q)}=1$$

$\therefore f(q)=1$

3. Let $x=r$

$$f(r)=\frac{(p-r)(q-r)}{(p-r)(q-r)}+0+0=1$$

$\therefore f(r)=1$

$\therefore f(x)=1$ at p, q, r

But if a quadratic function equals the same value at 3 distinct points, it must be a constant polynomial of that value.

Therefore, $f(x)=1$ for all x

Why a quadratic is a constant polynomial of a value if it equals a at three different points:

A quadratic polynomial is defined in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.

If $f(x) = 1$ at three different points, we can create a new polynomial $g(x)$, where

$$g(x) = f(x) - 1$$

$g(x)$ is still a quadratic, because it is equal to a quadratic ($f(x)$) minus a constant.

$g(x) = 0$ at $x = p, q, r$, because $f(x) = 1$ for all of those x values,

and $g(x) = f(x) - 1$

$$\therefore\ g(p) = 1 - 1 = 0$$
$$g(q) = 1 - 1 = 0$$
$$g(r) = 1 - 1 = 0$$

But a non zero quadratic polynomial cannot have more than 2 roots.
Therefore, $g(x)$ is the zero quadratic polynomial ($g(x) = 0$ for all x).

$$\therefore\ g(x) = f(x) - 1$$
$$0 = f(x) - 1$$
$$\therefore\ f(x) = 1 \text{ for all } x$$