

Proposed solutions to problems on exponential and logarithmic functions, with extension exercises

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Problem 1

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

A calculator is required in this question – give your answers in scientific notation.

- A. The mass of a neutron is about 1.675×10^{-27} kg. About how many neutrons are there in 1 kg of neutrons?

$$\begin{aligned} \text{a) } m &= 1.675 \times 10^{-27} \text{ kg} \\ \text{In 1 kg of neutrons,} \\ N &= \frac{1}{1.675 \times 10^{-27}} \\ &= \frac{1}{1.675} \times 10^{27} \quad \left\{ \frac{1}{1.675 \times 10^{-27}} = \frac{1}{1.675} \times \frac{1}{10^{-27}} \right\} \\ &\approx 0.597 \times 10^{27} \quad \left\{ \text{since the question uses 3 d.p.} \right\} \\ N &\approx 5.97 \times 10^{26} \quad \left\{ \begin{array}{l} \text{proper scientific notation is} \\ \text{a number between 1} \\ \text{and 10 multiplied by} \\ \text{a power of 10} \end{array} \right\} \\ &\text{neutrons.} \end{aligned}$$

- B. The radius of a neutron is about $1.11 \times 10^{-15} \text{ m}$. Use the formula for the volume of a sphere $V = \frac{4}{3}\pi r^3$ to find its approximate volume.

$$b) \quad r = 1.11 \times 10^{-15} \text{ m}$$

$$V = \frac{4}{3} \pi (1.11 \times 10^{-15})^3 \text{ m}^3$$

$$= \left[\frac{4}{3} \pi (1.11)^3 \right] \times 10^{-45} \text{ m}^3 \quad \left\{ \begin{array}{l} \text{continuing to} \\ \text{write units so} \\ \text{that I don't} \\ \text{forget!} \end{array} \right.$$

$$\approx 5.72 \times 10^{-45} \text{ m}^3$$

{calculated together rather than individually for accuracy}

- C. Use the formula density = mass volume to find its approximate density in kg/m^3 .

$$c) \quad \rho = \frac{m}{V} \quad \left\{ \begin{array}{l} \text{'rho' is 'rho', a greek} \\ \text{letter used to represent} \\ \text{density in physics} \end{array} \right.$$

$$\rho = \frac{1.675 \times 10^{-27}}{5.72 \times 10^{-45}} \text{ kg/m}^3$$

$$= \frac{1.675}{5.72} \times (10^{-27} \div 10^{-45}) \text{ kg/m}^3$$

$$\approx 0.293 \times 10^{18} \text{ kg/m}^3$$

$$\approx 2.93 \times 10^{17} \text{ kg/m}^3$$

Extension 1

- D. The mass of a neutron star is $2.0 \times 10^{30} \text{ kg}$. Using the density calculated in (C), estimate the volume of a neutron star.
- E. Assuming that the neutron star is a sphere, estimate its radius in kilometres.

Problem 2

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

Solve for x ,

$$\log_x 2 = -\frac{1}{4}$$

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$$x) \log_x 2 = -\frac{1}{4}$$

$$x^{-\frac{1}{4}} = 2 \quad \left\{ \begin{array}{l} \text{convert to index} \\ \text{equation} \end{array} \right\}$$

$$(x^{-\frac{1}{4}})^{-4} = 2^{-4} \quad \left\{ \begin{array}{l} \text{raise both sides} \\ \text{to the power} \\ \text{of } -4 \text{ to} \\ \text{express as } x \end{array} \right\}$$

$$x = \frac{1}{2^4}$$

$$x = \frac{1}{16}$$

Problem 3

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

Simplify,

$$2^{x \log_2 x}$$

$$2^{x \log_2 x} = 2^{\log_2 x^x} = x^x$$

$\left\{ \log_a a^n = n \log_a a \right\}$

$\left\{ \begin{array}{l} \text{the exponent } n \\ \text{that makes} \\ 2^n = x^x \end{array} \right\}$

A log is an indice! I tend to forget sometimes and think of them as base numbers, but reminding myself that they are indices forms a much clearer picture re the problem.

Extension 2

Simplify,

$$4^{\frac{x}{2} \log_2 x^{\frac{2}{3}}}$$

Problem 4

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

a. Solve $2^x < 10^{10}$

$$2^x < 10^{10}$$

$$\log_2 2^x < \log_2 10^{10} \quad \left\{ \text{take the } \log_2 \text{ of both sides} \right\}$$

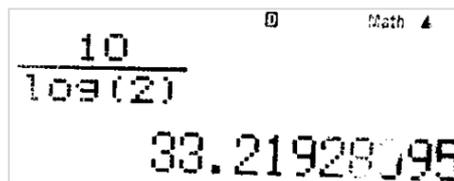
$$x < \log_2 10^{10} \quad \left\{ \text{that gets us } x \right\}$$

$$x < \frac{\log_{10} 10^{10}}{\log_{10} 2} \quad \left\{ \text{changing to base 10 using} \right.$$

$$\therefore x < \frac{10}{\log 2} \quad \left. \log_a x = \frac{\log_b x}{\log_b a}, \text{ to simplify numerator} \right\}$$

b. How many positive integer powers of 2 are less than 10^{10} ?

$$\frac{10}{\log 2} \approx 33.22 \text{ (2.d.p.)}$$



10
log(2)
33.21928095

$$\therefore x < 33 \quad \left\{ \text{the Q asks for integers} \right\}$$

\therefore 33 positive integer powers of 2 are less than 10^{10}

Extension 3

Let $N = 2^n$,

- A. Show that the number of digits of N is $(n \log_{10} 2) + 1$
- B. Find all integers for n for which 2^n has exactly 100 digits
- C. Hence, determine how many powers of 2 have exactly 100 digits

Problem 5

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

Find $\log_{10} 2^{1000}$, and hence find the number of digits in 2^{1000} .

$$\begin{aligned}\log_{10} 2^{1000} &= \frac{\log_2 2^{1000}}{\log_2 10} && \left\{ \begin{array}{l} \text{change of base to} \\ \text{get integer numerator} \end{array} \right\} \\ &= \frac{1000}{\log_2 10} && \left\{ \begin{array}{l} \text{cannot simplify} \\ \text{helpfully further} \end{array} \right\}\end{aligned}$$

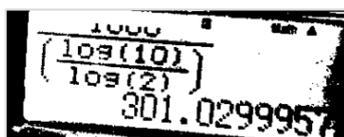
$$\begin{aligned}\text{Let } 2^{1000} &= x && \left\{ \begin{array}{l} \text{because } 2^{1000} \text{ is} \\ \text{just a number which} \\ \text{we don't know the} \\ \text{no. of digits of} \end{array} \right\} \\ \log_{10} x &= \frac{1000}{\log_2 10}\end{aligned}$$

$$x = 10^{\frac{1000}{\log_2 10}}$$

$\left\{ \begin{array}{l} \text{rewrite as index} \\ \text{equation and,} \\ \text{helpfully, our} \\ \text{base is 10} \end{array} \right\}$

Now,

$$\frac{1000}{\log_2 10} \approx 301$$



$$\therefore x \approx 10^{301}$$

$\left\{ \begin{array}{l} \text{which means the} \\ \text{number } 2^{1000} \text{ is approx.} \\ 10^{301}, \text{ or } 1 \text{ with } 301 \\ \text{zeroes} \end{array} \right\}$

$\therefore 2^{1000}$ has 302 digits $\left\{ 1 + 301 \text{ zeroes} \right\}$

Extension 4

Let $N = 3^{600} \times 2^{400}$.

Determine the number of digits in N .