

Proposed solutions to problems on lines in the coordinate plane, with extension exercises

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Grade 11 Mathematics
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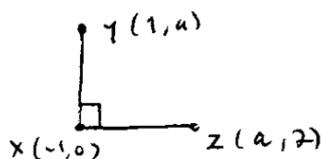
Contents

3	Problem 1
4	Extension 1
5	Problem 2
6	Extension 2
7	Problem 2
12	Extension 3

Problem 1

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

Given the points $X(-1, 0)$, $Y(1, a)$ and $Z(a, 2)$, find a if $\angle YXZ = 90^\circ$



$\angle YXZ = 90^\circ$
 $\therefore XY$ and XZ are perpendicular.

$$\therefore m_{XY} \times m_{XZ} = -1 \quad \left\{ \begin{array}{l} \text{perpendicular lines} \\ \text{condition} \end{array} \right.$$
$$\frac{(y_x - y_y)}{(x_x - x_y)} \times \frac{(y_z - y_x)}{(x_z - x_x)} = -1$$

$$\frac{(a - 0)}{(-1 - 1)} \times \frac{(2 - 0)}{(a - (-1))} = -1 \quad \left\{ \begin{array}{l} \text{had to be careful} \\ \text{with signs here} \end{array} \right.$$

$$\frac{a}{2} \times \frac{2}{a+1} = -1$$

$$\frac{a}{2} \times \frac{2}{a+1} = \frac{-1}{1} \quad \left\{ \begin{array}{l} \text{rewrite } -1 \text{ as} \\ \text{fraction} \end{array} \right.$$

$$2a = -1 \quad \left\{ \begin{array}{l} \text{equal numerators} \end{array} \right.$$

$$a = -\frac{1}{2}$$

Extension 1

- A. Find all values of a such that the triangle XYZ is right-angled.
Note that the right angle could occur at any vertex in this case.
- B. Find all values of A such that triangle XYZ has maximum area

Problem 2

Source: CambridgeMATHS NSW Stage 6 – Mathematics Advanced

Consider the two lines $l_1: 3x + 2y + 4 = 0$ and $l_2: 6x + \mu y + \lambda = 0$.

A. Find the value of μ if l_1 is parallel to l_2 .

$$l_1: 3x + 2y + 4 = 0$$

$$2y = -3x - 4$$

$$y = -\frac{3}{2}x - 2$$

$$l_2: 6x + \mu y + \lambda = 0$$

$$\mu y = -6x + \lambda$$

$$y = -\frac{6}{\mu}x + \frac{\lambda}{\mu}$$

$$-\frac{3}{2} = -\frac{6}{\mu}$$

$$-\frac{6}{4}$$

$$\therefore \mu = 4$$

Notes:

1. Convert both to gradient-intercept form, $y = mx + c$
2. If l_1 is parallel to l_2 , they have equal gradients
3. Therefore, the gradient of l_1 should be equal to the gradient of l_2
4. Setting the two gradients equal to each other leaves μ as the only variable, and we can solve

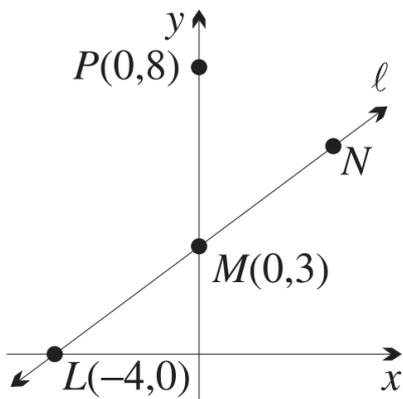
Extension 2

- B. Using your answer to (A), find the value of λ such that the triangle formed by ι_2 and the coordinate axis has minimum area.
- C. Determine the minimum area of the triangle in (B)

Problem 3

Source: *CambridgeMATHS NSW Stage 6 – Mathematics Advanced*

The line l crosses the x - and y -axes at $L(-4, 0)$ and $M(0, 3)$. The point N lies on l , and P is the point $P(0, 8)$.



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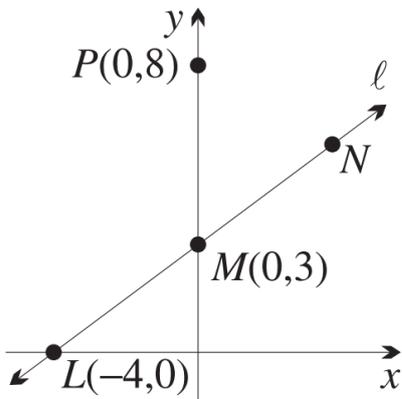
- a. Copy the sketch and find the equation of l .

$$m_l = \frac{(y_M - y_L)}{(x_M - x_L)} = \frac{(3 - 0)}{(0 + 4)} = \frac{3}{4}$$

$$\therefore l_1 : \frac{3}{4}x + 3$$

Notes:

1. Find the gradient of l using our two known points M and L
2. Conveniently, M is the y -intercept of l , so we can write $y = mx + c$ form



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- b. Find the lengths of ML and MP and hence show that LMP is an isosceles triangle

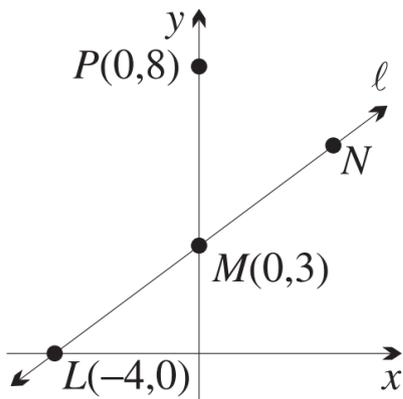
$$\begin{aligned}
 ML^2 &= (x_M - x_L)^2 + (y_M - y_L)^2 \\
 &= (0 + 4)^2 + (3 - 0)^2 \\
 &= 16 + 9 \\
 &= 25 \\
 ML &= \sqrt{25} = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{Now,} \\
 PM^2 &= (x_P - x_M)^2 + (y_P - y_M)^2 \\
 &= (0 - 0)^2 + (8 - 3)^2 \\
 &= 25 \\
 PM &= \sqrt{25} = 5
 \end{aligned}$$

$\triangle LMP$ is a triangle
 $PM = ML$
 $\therefore \triangle LMP$ is isosceles

Notes:

1. Use the distance formula $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ to find the lengths of ML and MP .



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c. If M is the midpoint of LN , find the coordinates of N .

$$x_M = 0 = \frac{x_N + x_L}{2} = \frac{x_N - 4}{2}$$

$$0 = x_N - 4$$

$$\therefore x_N = 4$$

$$y_M = 3 = \frac{y_N + y_L}{2} = \frac{y_N + 0}{2}$$

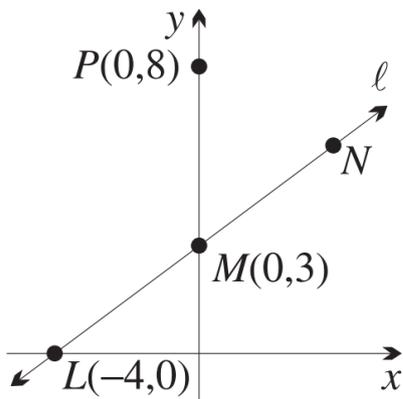
$$\therefore 6 = y_N$$

$$\therefore N(x_N, y_N)$$

$$N(4, 6)$$

Notes:

1. Since we are given the midpoint and one endpoint, we can use the midpoint formula $x = \frac{x_1 + x_2}{2}$ (and vice versa for y) in reverse to find the other endpoint.
2. The x, y coordinates of a midpoint are just the average of the x, y coordinates of its endpoints.



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d. Show that $\angle NPL = 90^\circ$.

$$m_{NP} = \frac{(y_N - y_P)}{(x_N - x_P)} = \frac{(6 - 8)}{(4 - 0)} = -\frac{2}{4} = -\frac{1}{2}$$

$$m_{PL} = \frac{(y_P - y_L)}{(x_P - x_L)} = \frac{(8 - 0)}{(0 + 4)} = 2$$

If $m_{NP} \times m_{PL} = -1$, NP and PL are perpendicular,

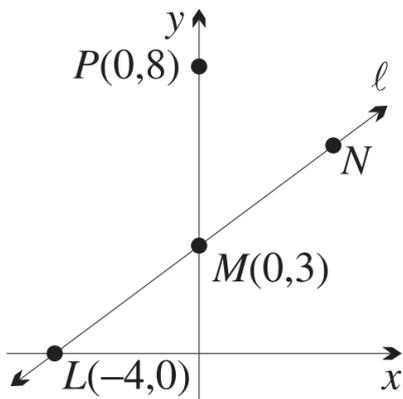
$$m_{NP} \times m_{PL} = -\frac{1}{2} \times 2 = -1$$

$\therefore NP$ and PL are perpendicular

$\therefore \angle NPL = 90^\circ$

Notes

1. Find the gradients of NP and PL , because we need them to use the perpendicular condition $m_1 \times m_2 = -1$ and prove that NP and PL are perpendicular.



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- e. Write down the equation of the circle through N, P and L

The circle has a centre $M(0,3)$
 { N and L must be the outer bounds, so their midpoint is the centre of the circle }

The circle has a radius $ML = 5$
 { From part (b), since M is the centre, the distance from M to outer bound L is the radius }

The circle passes through $P(0,8)$

Using the circle centre equation
 $(x-h)^2 + (y-k)^2 = r^2$, where
 (h,k) is the circle's centre and
 r is its radius,

$$\begin{aligned} (x - x_M)^2 + (y - y_M)^2 &= ML^2 \\ (x)^2 + (y - 3)^2 &= 25 \\ x^2 + y^2 - 6y + 9 &= 25 \\ x^2 + y^2 - 6y - 16 &= 0 \end{aligned}$$

{ General form of the equation of a circle }

$$\{ x^2 + y^2 + Dx + Ey + F = 0 \}$$

Extension 3

- f. Find the equation of the tangent to the circle in part (e) at the point N .
- g. Find the gradient of the tangent in (f)
- h. Another point Q lies on the circle such that PQ is parallel to LM .
Find the coordinates of Q .