Theory on the sine function and its amplitude

Tolentino Tuition
Modbury Heights, tolentinotuition.com
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Contents

What does the sine function represent?

It is helpful to think about what the sine function actually represents before dealing with it in calculations.

First, *what is a function*?

It's simply an *expression* where you can *input* a value, and get an *output*:

$$\text{e.g. } y = x^2$$

is a function, because you can *input* a value for x:

$$y = (3)^2$$

And the expression will give you an output, which is y in this case:

$$y = 3^2$$
$$y = 9$$

The *sine function*, is no different!

You can input a value, usually denoted by x or θ (theta):

$$y = \sin\theta$$

$$y = \sin\frac{3\pi}{4}$$

And get an output, which is y:

$$y = \frac{\sqrt{2}}{2}$$

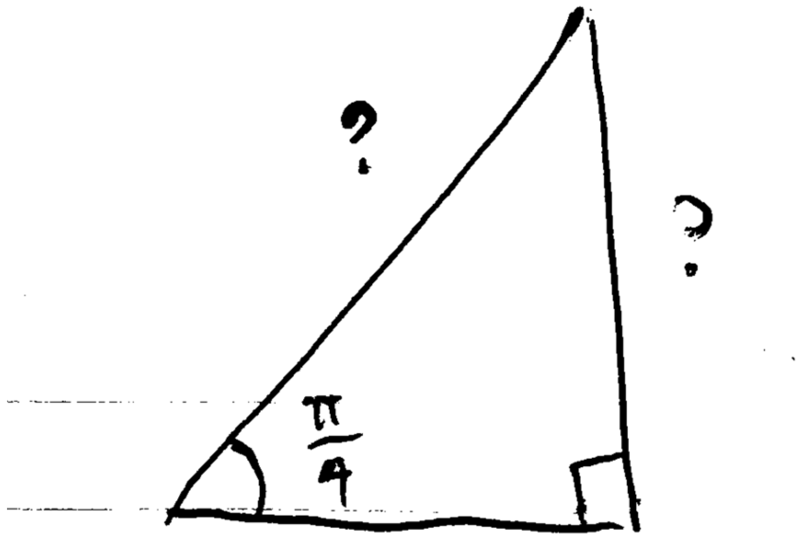
Now that is wonderful, but it's more helpful to know *why* a *sine function* is actually useful:

> For any right angled triangle, if you *input* one of the angles that you know (that isn't the right angle) into the *sine function* as θ (theta), the function will tell you the *ratio* between the side of the right angled triangle which is opposite to θ, and the hypotenuse...

Now that's a lot of words – it is much easier to represent this visually!

So, say that someone told you that one of the angles in a right angled triangle is $\frac{\pi}{4}$ ...

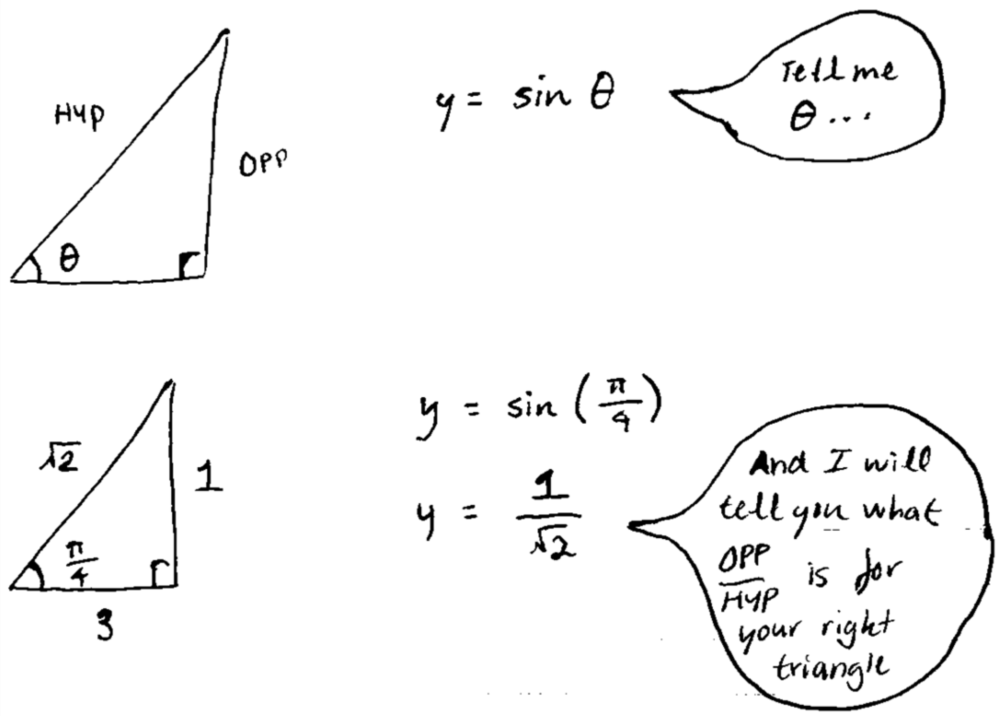
But for whatever reason, you wanted to know what the *ratio* between the side of the right angled triangle which is opposite to that $\frac{\pi}{4}$ angle, and the hypotenuse...



With the *ratio* meaning, 'how big is the *opposite side* compared to the hypotenuse?'

(For example, the *ratio* $\frac{1}{2}$ tells us how big the number 1 is compared to the number 2. $\frac{1}{2} = 0.5$, meaning that 1 is half as big as two!)

The *sine* function can give you this ratio! All you have to do is give it that one angle in the right angle that you know, $\frac{\pi}{4}$ in this case...



So, we *inputted* $\frac{\pi}{4}$ into the $sine\ function$, and it *outputted* for us the *ratio* of the *opposite* and *hypotenuse* sides of a right angled triangle which has one of its angles as $\frac{\pi}{4}$ – that's very cool! Also note, that the function represents this ratio as y, which is why we end up with $y =$

How to think about the graph of a sine function

As with other functions, we can make a $graph$ of a sine function.

But what's the point of doing so?

Let's think about it this way...

Say that someone tells me to picture two right angled triangles...

Triangle 1 has one of its angles as $\frac{\pi}{3}$, and Triangle 2 has one of its angles as $\frac{\pi}{6}$.

Now, if I wanted to know the ratio of $\frac{OPP}{HYP}$ for both of these right angled triangles, I could plug them into the $sine\ function$ and get my answers...

$$\text{Triangle 1: } y = sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

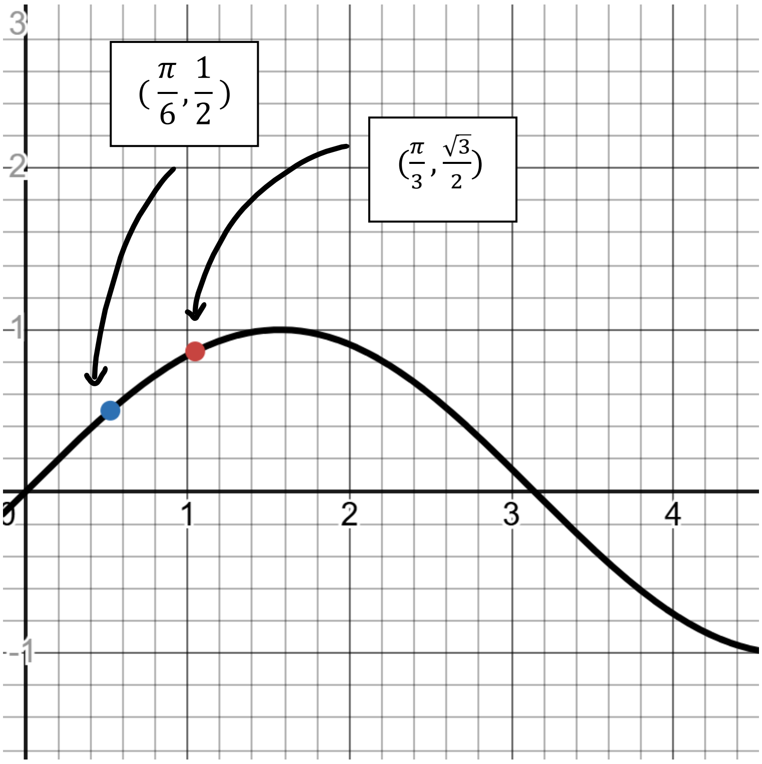
$$\text{Triangle 2: } y = sin\frac{\pi}{6} = \frac{1}{2}$$

Now that's wonderful, but what if my friend gave me 7 triangles to find the $\frac{OPP}{HYP}$ ratios for...

I could do all seven calculations myself, but that would be tedious.

If only there were a way to represent all the possible values of the ratio y depending on what value of θ I gave the $sine$ function...

That's why we use graphs! (below: $y = \sin\theta$)



The (θ, y) coordinates of the blue point are $(\frac{\pi}{6}, \frac{1}{2})$, which shows us that if one of the angles in a right angled triangle is $\frac{\pi}{6}$, the triangle's $\frac{OPP}{HYP}$ ratio will be $\frac{1}{2}$

And the (θ, y) coordinates of the red point are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$, which shows us that if one of the angles in a right angled triangle is $\frac{\pi}{3}$, the triangle's $\frac{OPP}{HYP}$ ratio will be $\frac{\sqrt{3}}{2}$

So what's wonderful about *graphing the sine function* is that I can now see as many possible *ratios* (y) as I want, depending on what θ is on the x axis, without having to do all the calculations myself!

How to think about amplitude

Now, think about this...

Say that every time I knew one angle in a right angled triangle, and wanted to calculate the $\frac{OPP}{HYP}$ ratio of my triangle using the $sine\ function$,

for whatever reason, after I calculated that ratio, I want to multiply that ratio by 2 ... every time

How might I adjust my function $\sin\theta$ so that I can do this?

Well, since $\sin\theta$ gives me the ratio, I could just multiply $\sin\theta$ by 2 ...

$$\sin\theta \times 2 = 2\sin\theta$$

Now, every time I *input* an angle, the $\sin\theta$ part of my function will calculate the ratio, whilst the $\times\ 2$ part will multiply that ratio by 2

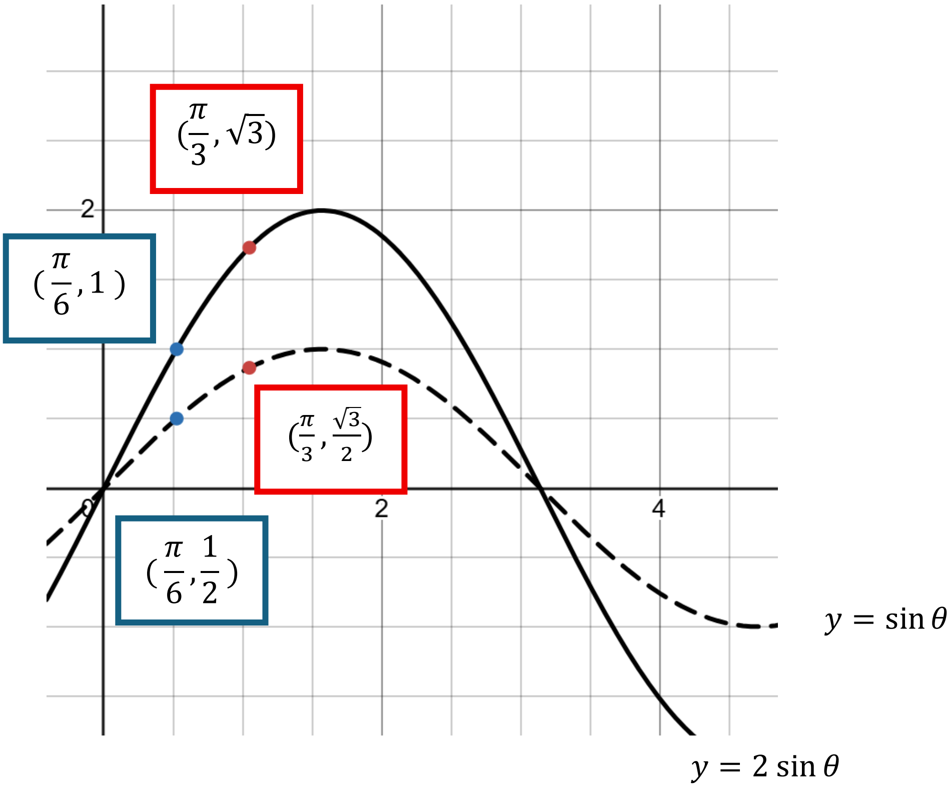
Before: $y = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\{$ simply getting the ratio $\}$

After: $y = 2\ \sin\frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$ $\{$ getting the ratio, then multiplying it by 2, pretty neat! $\}$

Before we finish, question for you…

Now that our function is $y = 2\sin\theta$, would its graph be different from that of $y = \sin\theta$?

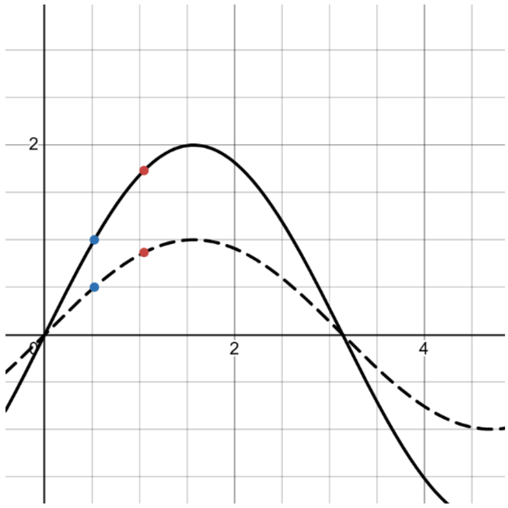
Well, let’s graph them both and see!



We can see that, for the same values of θ (which is simply that one angle in the R.A triangle that we know, if you remember), the function $y = 2\sin\theta$ gives us double what the ratio was in $y = \sin\theta$. Which is want we wanted!

You may also notice that the graph of $y = 2\sin\theta$ is quite a bit taller than $y = \sin\theta$. It’s twice as tall at every point to be exact! Because we are now multiplying the ratio by 2 every time.

We call the length from the θ axis to the maximum y value of $\sin\theta$ its 'amplitude'. And you will notice that the amplitude of $y = \sin\theta$ is 1, whilst it is 2 for $y = 2\sin\theta$.



This shows two profoundly interesting things:

1. Because the amplitude or maximum y value of $y = \sin\theta$ is 1, and y represents the $\frac{OPP}{HYP}$ ratio in a right angled triangle, this means that the maximum *ratio* of the $\frac{OPP}{HYP}$ in a right angled triangle, regardless of what its angles are, is 1:1 $= 1$. Meaning, the opposite side and hypotenuse are the same length.

   Which makes sense right? Because by definition the *hypotenuse* is the longest side of a triangle. So, in our $\frac{OPP}{HYP}$ ratio, the denominator, representing the hypotenuse, *can never be smaller than the numerator*, otherwise that side wouldn't be the hypotenuse!

2. If we multiply the function $y = \sin\theta$ by a number, that number will become the amplitude of its graph, because the amplitude or maximum y value of $y = \sin\theta$ is 1, and $1 \times$ any number equals that number!

   That's why $y = 2\sin\theta$ has an amplitude of 2, and why $y = \frac{1}{2}\sin\theta$ would have an amplitude of $\frac{1}{2}$, and so on...