Grade 11 Advanced Mathematics revision exercises and proposed solutions

Tolentino Tuition
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Contents

Exercise 1

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1. Given that the distance d between two points on a cartesian plane is

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, prove that

   A. The equation of a circle centred at the origin with radius r is given by $x^2 + y^2 = r^2$.

   B. The equation of a circle centred at (h, k) with radius r is given by $(x - h)^2 + (y - k)^2 = r^2$.

2. Express the equation of the circle in (B) such that the circle

   A. is shifted $\frac{1}{2}$ of a unit upwards

   B. is shifted π units downwards and 3 units right

   C. is shifted 2 units left and has a radius equal to the x coordinate of its centre

   D. is shifted $\frac{3}{2}$ units right and has a radius proportional to the y coordinate of its centre

Exercise 2

Source: *CambridgeMATHS NSW Stage 6 Advanced Year 11 2e*

1. Rationalise the denominator of

$$\frac{3\sqrt{3}+5}{3\sqrt{3}-5}$$

2. Factor the numerator and denominator where possible, then simplify

$$\frac{3x^2-19x-14}{9x^2-4}$$

3. Find the value of x if $\sqrt{18}+\sqrt{8}=\sqrt{x}$

4. Find the values of p and q such that $\frac{\sqrt{5}}{\sqrt{5}-2}=p+q\sqrt{5}$

Exercise 3

Source: $CambridgeMATHS\ NSW\ Stage\ 6\ Advanced\ Year\ 11\ 2e$

1. Find the values of α that will make $(\alpha + 6)x^2 - 2\alpha x + 3$ a perfect square

2. A farmer needs to build a rectangular holding pen for some sheep.
   The side of a barn will be used for one side of the pen and there is 60 m of fencing available for the remaining three sides.
   Let x be the length of each side perpendicular to the barn wall.

   A. Find an expression for the third side in terms of x and draw a diagram showing all this information.

   B. The area A of the pen must be at least 400 m^2 so that the sheep are not overcrowded. What is the smallest value of x for these requirements?

   Challenge: Solve for x whilst maintaining units in your calculations.

Exercise 4

Source: $CambridgeMATHS\ NSW\ Stage\ 6\ Advanced\ Year\ 11\ 2e$

1. Determine whether the lines $8x + 7y + 6 = 0$, $6x - 4y + 3 = 0$ and $2x + 3y + 9 = 0$ enclose a right-angled triangle.

2. When an object is thrown up a hill, its maximum range R is the greatest distance it can reach.
   The maximum range of an object varies directly with the square of the speed V it is thrown at.

   A. Write this algebraically using k as the constant of proportionality.

   B. It is found that $R = 5.505\ m$, correct to four significant figures, when $V = 9ms^{-1}$. Evaluate k correct to four significant figures.

   C. What is R when $V = 15\ ms^{-1}$? Give your answer correct to the nearest centimetre.

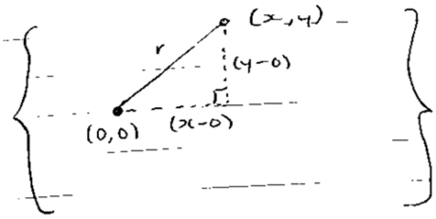
Proposed solutions

Exercise 1

Exercise 1

The centre of the circle is (0, 0).
Its radius (r) is the distance between its centre (0, 0) and some point (x, y).

Hence,
$r^2 = (x-0)^2 + (y-0)^2$



$r^2 = x^2 + y^2$

Centre at (h, k)
r is the distance between centre (h, k) and some point (x, y)

Hence,
$r^2 = (x-h)^2 + (y-k)^2$

The centre is (h, k), adjusting k will move the circle vertically:

(A) $(x-h)^2 + \left(y - \left(k + \frac{1}{2}\right)\right)^2 = r^2$

(B) $(x-(k+3))^2 + (y-(k-\pi))^2 = r^2$

(C) $(x-(k-2))^2 + (y-k)^2 = (k-2)^2$

(D) $\left(x-\left(k+\frac{3}{2}\right)\right) + (y-k)^2 = kr^2$

Exercise 2 (1) and (2)

Exercise 2

1) $$\frac{3\sqrt{3}+5}{3\sqrt{3}-5} = \frac{(3\sqrt{3}+5)^2}{(3\sqrt{3}-5)(3\sqrt{3}+5)}$$

$$= \frac{(3\sqrt{3}+5)^2}{(3\sqrt{3})^2-(5)^2} \quad \{(a+b)(a-b) = a^2-b^2\}$$

$$= \frac{(3\sqrt{3}+5)^2}{27-25} = \frac{(3\sqrt{3}+5)^2}{2}$$

(2) $$\frac{3x^2-19x-14}{9x^2-4} = \frac{3x^2+2x-21x-14}{(3x)^2-(2)^2} \quad \left\{\begin{array}{l} 3x-14 = -42 \\ 2x-21 = -42 \\ 2-21 = -19 \end{array}\right.$$

$$= \frac{x(3x+2)-7(3x+2)}{(3x+2)(3x-2)} \quad \{(a+b)(a-b) = a^2-b^2$$

$$= \frac{(3x+2)(x-7)}{(3x+2)(3x-2)}$$

$$= \frac{x-7}{3x-2}$$

Exercise 2 (3)

$$\sqrt{18} + \sqrt{8} = \sqrt{x}$$
$$(\sqrt{18} + \sqrt{8})^2 = (\sqrt{x})^2$$
$$(\sqrt{18})^2 + 2\sqrt{18} \cdot \sqrt{8} + (\sqrt{8})^2 = x$$
$$18 + 2\sqrt{144} + 8 = x$$
$$18 + 2 \cdot 12 + 8 = x$$
$$x = 50$$

Exercise 2 (4)

$$\frac{\sqrt{5}}{\sqrt{5} - 2} = \frac{\sqrt{5}(\sqrt{5} + 2)}{(\sqrt{5} - 2)(\sqrt{5} + 2)}$$
$$= \frac{(\sqrt{5})^2 + 2\sqrt{5}}{(\sqrt{5})^2 - (2)^2}$$
$$= \frac{5 + 2\sqrt{5}}{5 - 4}$$
$$= 5 + 2\sqrt{5}$$

$$\therefore p = 5, \ q = 2$$

Exercise 3 (1)

$$(a+6)x^2 - 2ax + 3 = x^2 + 2bx + b^2$$

$$x^2 - \frac{2ax}{(a+6)} + \frac{3}{(a+6)} = x^2 + 2bx + b^2$$

$$x^2 - 2\left(\frac{a}{a+6}\right)x + \frac{3}{(a+6)} = x^2 + 2bx + b^2$$

$$\therefore \quad \frac{a}{a+6} = b \quad \text{and} \quad \frac{3}{(a+6)} = b^2$$

$$\{(1)\} \qquad \{(2)\}$$

$$\left(\frac{a}{a+6}\right)^2 = b^2$$

$$\frac{a^2}{(a+6)^2} = \frac{3}{(a+6)} \quad \{(2) \text{ into } (1)\}$$

$$\frac{a^2}{(a+6)^2} = \frac{3(a+6)}{(a+6)^2}$$

$$a^2 = 3(a+6)$$

$$= 3a + 18$$

$$a^2 - 3a - 18 = 0$$

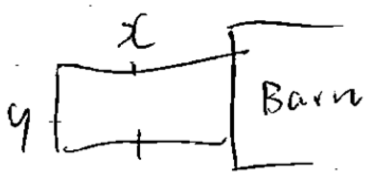
$$a^2 + 3a - 6a - 18 = 0$$

$$a(a+3) - 6(a+3) = 0$$

$$(a+3)(a-6) = 0$$

$$a = -3, 6$$

Exercise 3 (2)



$2x + y = 60$

$y = 60 - 2x$

$A = x(60 - 2x)$ (1)

$A \geq 400\ m^2$

$x(60 - 2x)\,m^2 \geq 400\ m^2$ (1)

$(60x - 2x^2)\,m^2 \geq 400\ m^2$

$(-2x^2 + 60x)\,m^2 - 400\ m^2 \geq 0$

$(-2x^2 + 60x - 400)\ m^2 \geq 0$

$(x^2 - 30x + 200)\ m^2 \geq 0$

$(x^2 - 10x - 20x + 200)\ m^2 \geq 0$

$(x(x - 10) - 20(x - 10))\ m^2 \geq 0$

$(x - 20)(x - 10)\ m^2 \geq 0$

$(x - 20)\,m\ (x - 10)\,m \geq 0$

$(xm - 20m)(xm - 10m) \geq 0$

$xm - 20m \geq 0$

$xm \geq 20m$

$xm - 10m \geq 0$

$xm \geq 10m$

smaller!

Exercise 4

1. Convert the three lines into slope intercept form: $y = mx + c$, to determine the slope of each line.

2. Recall that the product of the slopes m_1 and m_2 of two lines will equal -1 if and only if the lines are perpendicular to each other,

   Ie. $m_1 \times m_2 = -1$

   Evaluate each combination of the products of the slopes for the three lines to determine whether any two are perpendicular.

   Why this is relevant is because the angle created by two perpendicular lines will be equal to 90 degrees, and we have been asked to determine whether our three lines enclose a right angled triangle. So, if we find that no dual combination of our three lines is perpendicular, then the three lines do not enclose a right angled triangle.

3. If we find that two of our three lines are perpendicular, we then need to set the equations of these two lines equal to each other

   Ie. $m_1x + c_1 = m_2x + c_2$

   to determine the x coordinate of the point at which they intersect, which we can substitute into any one of the equations of the two perpendicular lines to confirm the y coordinate at their point of intersection.

   We then set the equation of the non-perpendicular line equal to the equations of the each of the two perpendicular lines separately, to determine the coordinates at which the non-perpendicular line intersects the two perpendicular lines.

   We can then graph our three lines and three intersection points to confirm whether or not the three lines enclose a right angled triangle.

Exercise 4 (2)

(A) $R = kV^2$