# PROPOSED SOLUTIONS TO PROBLEMS ON CUBIC POLYNOMIALS FOR S.R.


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# QUESTION 9(a)

9. (a) Find the value of $m$ if $(x+3)$ is a factor of the cubic: $2x^3 - 15x^2 + mx - 21$ **(4 marks)**

There are two ways to do this!

The first is quicker, but I recommend the second, as the second involves calculations that can aid us in **Question 9(b).**

1. **Using the 'factor theorem'**
   Since we know that $(x+3)$ is a factor of $2x^3 - 15x + mx - 21$ (which we will call $P(x)$), we know that $x = -3$ will be a zero/solution to $P(x)$ – meaning – if we substitute $-3$ into $P(x)$, $P(x)$ will equal zero!

$(x+3)$ is a factor of $2x^3 - 15x^2 + mx - 21$

Factor Theorem: If $(x-c)$ is a factor of $P(x)$, then $P(c) = 0$

$\therefore (x+3) = (x-c)$, $c = -3$

$\therefore P(-3) = 0$

$P(-3) = 2(-3)^3 - 15(-3)^2 + m(-3) - 21 = 0$

$= 2(-27) - 15(9) - 3m - 21 = 0$

$= -54 - 135 - 3m - 21 = 0$

$= -210 - 3m = 0$

$= -3m = +210$

$= m = \frac{210}{-3}$

$m = -70$

2. **A linear factor multiplied by a quadratic equals a cubic polynomial**
Since we know that $(x + 3)$ is a linear factor of $P(x)$, and we know that $P(x)$ is a cubic polynomial, we also know that some quadratic function $ax^2 + bx + c$ exists such that
$(x + 3)(ax^2 + bx + c) = 2x^3 - 15x^2 + mx - 21$
Because a linear function (a function where the highest degree is $x^1$) multiplied by a quadratic function (highest degree $x^2$) equals a cubic function ($x^1 \times x^2 = x^{1+2} = x^3$).

$(x+3)$ is a factor of $2x^3 - 15x^2 + mx - 21$

$\therefore (x+3)(ax^2 + bx + c) = 2x^3 - 15x^2 + mx - 21$

$ax^3 + 3ax^2 + bx^2 + 3bx + cx + 3c = \cdots$

$ax^3 + (3a+b)x^2 + (3b+c)x + 3c = \cdots$

$\{ ax^3 = 2x^3, \therefore a = 2 \}$

$2x^3 + (6+b)x^2 + (3b+c)x + 3c = \cdots$

$\{ +(6+b)x^2 = -15x^2, 6+b = -15, \therefore b = -21 \}$

$2x^3 - 15x^2 + (3(-21) + c)x + 3c = \cdots$

$2x^3 - 15x^2 + (c - 63)x + 3c = \cdots$

$\{ +3c = -21, \therefore c = -7 \}$

$2x^3 - 15x^2 + ((-7) - 63)x - 21 = \cdots$

$2x^3 - 15x^2 - 70x - 21 = 2x^3 - 15x^2 + mx - 21$

$-70x = mx$

$m = -70$

QUESTION 9(b)

(b) Hence find all solutions of the cubic equation: $2x^3 - 15x^2 + mx - 21 = 0$ (2 marks)

Find all solutions to $2x^3 - 15x^2 - 70x - 21$

From (a) we know that:

$2x^3 - 15x^2 - 70x - 21 = (x+3)(ax^2+bx+c)$

And also that $a = 2$, $b = -21$, $c = -7$:

$\therefore\ 2x^3 - 15x^2 - 70x - 21 = (x+3)(2x^2 - 21x - 7)$

Remember that solutions to cubics are easiest to find when they are in this form: $(x-a)(x-b)(x-c)$

If we factorise $(2x^2 - 21x - 7)$ using the quadratic formula:

$2x^2 - 21x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{21 \pm \sqrt{(-21)^2 - 4(2)(-7)}}{4}$$

$$= \frac{21 \pm \sqrt{441 + 56}}{4}$$

$$\therefore x = \frac{21 - \sqrt{497}}{4} \quad \text{or} \quad \frac{21 + \sqrt{497}}{4}$$

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Find all solutions to $2x^3-15x^2-70x-21$

From (a) we know that:

$2x^3-15x^2-70x-21 = (x+3)(ax^2+bx+c)$

And also that $a=2$, $b=-21$, $c=-7$:

$\therefore 2x^3-15x^2-70x-21 = (x+3)(2x^2-21x-7)$

Remember that solutions to cubics are easiest to find when they are in this form: $(x-a)(x-b)(x-c)$

If we factorise $(2x^2-21x-7)$ using the quadratic formula:

$2x^3-15x^2-70x-21 = (x+3)\left(x-\left(\frac{21+\sqrt{497}}{4}\right)\right)\left(x-\left(\frac{21-\sqrt{497}}{4}\right)\right)$

$\therefore x = \frac{21-\sqrt{497}}{4}$, $-3$ and $\frac{21+\sqrt{497}}{4}$

$\approx -0.32$, $-3$ and $10.82$